This paper is based on my dissertation which explores mathematical beliefs, a field that has been discussed internationally more than ten years. It looks for answers to the questions it addresses especially from the perspective of Finnish primary school teachers, from the perspective of the first and the second grade’s teaching culture. This paper focuses on primary school teachers’ mathematics-related beliefs, which pertain to their subjective knowledge. These beliefs act in mathematics classrooms as a hidden factor regulating the quality of mathematics teaching and learning. Some examples of teachers’ beliefs and their teaching practices are exposed and discussed.

Introduction

In Finland primary school teachers work in grades 1-6 where pupils are from 7 to 12 years old. Teachers are ‘class teachers’, meaning that they have to be able to teach all subjects, including for example religious knowledge, mother tongue, music, arts etc. They have to teach about ten subjects. In grades 7 - 9 mathematics teachers teach mathematics and also other subjects have their own teachers.

The study attempts to understand and describe instructional practices of 7- to 8-year-old children’s teachers, especially the use of textbooks, and the preconditions under which these practices develop and change.

This paper is based on the qualitative part of my dissertation study. This part of the study describes instructional practices and teacher’s beliefs. In this paper I describe how they teach mathematics in the first and the second grade in primary school. What is the role of teachers’ beliefs during the mathematics lessons?

The view of mathematics

Knowledge is often separated in two parts: objective knowledge and subjective knowledge. Objective knowledge is scientific knowledge, which can be explicated and discussed. For example in mathematics it means the generally accepted structure of mathematics, which is a compound of all mathematicians’ research work from more than 2000 years. (Pehkonen 2001). According to Pietilä (2002) beliefs can be individual’s subjective knowledge or both individual’s subjective knowledge and his feelings. A belief can merely be subjective knowledge when a pupil thinks that he knows something (for example how to divide by a 2-digit number) and he acts on his belief. A pupil can also believe that he is not good at mathematics and that is the reason why he cannot learn mathematics. This kind of belief has feelings with no success. (Pietilä 2002.) There can also be other mathematics-related beliefs such as ‘you
can learn mathematics only at school’, ‘mathematics is difficult’. ‘There are only few people who can learn mathematics; ‘That is why only few people can do mathematics.’; ‘mathematics is abstract and it has no connection with everyday life and that is why you can not learn it in everyday life’. (see Nunes & Bryant 1996.)

According to Abelson (1979) beliefs are usually held with a different degree of conviction. Beliefs can be understood as non-conscious and conscious beliefs. We can define conscious beliefs as conceptions. According to Saari (1983), we can understand conceptions as a subset of beliefs. Here individual’s beliefs are understood to be composed of his subjective experience-based implicit knowledge of mathematics and its teaching and learning. The beliefs - conscious and unconscious - can be seen as a belief system.’ When the object of the belief system is mathematics or mathematics teaching / learning I use the term: ‘view of mathematics’.

Ernest (1989) has distinguished three different conceptions of the nature of mathematics, which answer the question “What is mathematics?“. These are ‘the instrumentalist view, the Platonist view, and the problem solving view’. The teacher’s view of mathematics is the basis for the teacher’s mental models of teaching and learning mathematics. When the teacher has an instrumentalist view of mathematics, ‘a typical feature of his teaching is the strict following of teaching models and, for example, the strict following of a textbook’s text and scheme. The teacher is a transmitter of knowledge and completed rules. Typical of this model is children’s compliant behaviour and mastery of skills model. The children are calculating and, utilizing rules and procedures. In the Platonic view of mathematics the role of the teacher can be seen as an explainer. This model is likely to be associated with conceptual understanding with unified knowledge. Children’s learning is still the reception of knowledge model. Thus mathematics is discovered, not created. In the problem-solving view of mathematics the teacher is a facilitator and a child is an active constructor of knowledge. This model is based on the child’s active construction of understanding model and exploration and autonomous pursuit of his own interests model.’ (Ernest 1989; see also Dionne 1984.)

Method

The basis of my data is a Likert-scale belief questionnaire. The Likert-scale questionnaire was sent to the first and the second grade teachers in different primary schools. This belief questionnaire included 70 statements concerning teachers’ beliefs and conceptions about mathematics, learning mathematics, teaching mathematics, and mathematics teaching practices, especially use of mathematics textbooks. For the classification I classified the statements of the questionnaire on the basis of their contents to the following four levels:

A. What is mathematics?

B. Mathematics learning in the first and second grade.

C. Mathematics teaching in the first and the second grade.
D. Mathematics teaching activities in the first and the second grade.

To make comparisons, the teachers’ answers (beliefs) were classified in each level on the following scale: traditional, primarily traditional, mixed, primarily non-traditional, and non traditional. As the basis of this classification there were Ernest’s (1989) three views of mathematics. The traditional answers were near the instrumentalist view of mathematics, and the non-traditional answers were near the problem solving view of mathematics. On the basis of this classification I selected 6 teachers (N=140) representing the first and the second grades from different primary schools.

The following table presents the beliefs of the six teachers. The beliefs are put in order on the basis of the first level A beliefs. In parenthesis there are arithmetical means of each beliefs.

Table 1. The beliefs of the six teachers about mathematics (A), mathematics learning (B), mathematics teaching (C) and mathematics teaching activities (D).

<table>
<thead>
<tr>
<th>Teacher/ level</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anna M*</td>
<td>M* (3,40)</td>
<td>M (2,85)</td>
<td>M (3,16)</td>
<td>M(3,13)</td>
</tr>
<tr>
<td>Bertta</td>
<td>M(3,00)</td>
<td>PNT (2,26)</td>
<td>M(2,66)</td>
<td>M(2,50)</td>
</tr>
<tr>
<td>Cecilia</td>
<td>PNT 2,35</td>
<td>PNT(1,93)</td>
<td>PNT(2,00)</td>
<td>PNT (1,97)</td>
</tr>
<tr>
<td>Doris</td>
<td>PNT (2,05)</td>
<td>M(2,70)</td>
<td>M(2,88)</td>
<td>M(2,91)</td>
</tr>
<tr>
<td>Enni</td>
<td>PNT(1,70)</td>
<td>PNT(1,78)</td>
<td>PNT(1,88)</td>
<td>PNT(2,25)</td>
</tr>
<tr>
<td>Fanni</td>
<td>PNT(1,60)</td>
<td>PNT(1,89)</td>
<td>PNT (2,06)</td>
<td>M(2,53)</td>
</tr>
</tbody>
</table>

* NT = non traditional (1), PNT = primarily non-traditional (2), M = mixed (3), PT = primarily traditional (4), T = traditional (5). The mean of the answers (scored in this way) is in parentheses.

Teaching experience of these 6 teachers varied from 2 to 30 years. Each teacher was contacted first by phone and all of them volunteered to participate in the study. Data collection for each teacher included: a) two weeks’ classroom observations (video-taped), b) teachers’ written lesson plans, c) an interview of the teachers after the classroom observation period (the questions during the interview focused on the nature of mathematics, mathematics teaching and learning practices and mathematics textbooks, assessment, school recollections about mathematics and teachers’ mathematics studies), d) analysis of the video-taped mathematics lessons together with the teacher.

**Results**

Notes from classroom observations and interviews of those 6 teachers were used to describe consistencies and inconsistencies between the teachers’ beliefs and teaching
practices. According to Ernest (1989), there are three key elements that influence the practice of mathematics teaching: ‘the powerful influence of the social context; the teacher’s level of consciousness of his own beliefs and the extent to which the teacher reflects on his practice of mathematics’. I found that teachers’ recollections of their experiences (e.g. difficulties with mathematics learning, their school-time teacher’s dependence on mathematics textbooks and inclination to the rules and routines methods) in mathematics at school influenced their teaching practices. The study of Lindgren (1998) gives support to this. In Lindgren’s study (1998) ‘special attention was paid to the emotional memories of the students’ math teachers. Her interviews with student teachers revealed hints for correlation between recalled math teachers’ harshness, sharpness, and dependence on mathematics schoolbooks with student teacher’s distrust of the open-approach method and inclination to the rules and routines method.’

The use of manipulatives in the teaching / learning situations was often regarded as useful for promoting the view that “math is fun”. For example teacher’s beliefs about mathematics were primarily non-traditional but his instructional practice was still focused on textbooks, rules, and procedures. There appeared inconsistencies between their beliefs and practices. Thompson (1984), Brown (1985) and Cooney (1985) have identified inconsistencies between the relationships of beliefs and classroom activities, too. They consider this relationship complicated because many social norms with possibilities and limits are connected: values, expectations and beliefs of pupils, parents, and administrators. The influence of the curriculum, the practices of assessment and the dominating views and values of learning are connected. Those factors mentioned above have an effect on teacher’s instruction.

Sometimes the teacher’s school-time recollections, e.g. difficulties with mathematics learning, were so deep that he wanted to protect the children - he did not give children space to think. Interviews and observations revealed that teachers should have more mathematics studies during their teacher education. Because of the uncertainty with mathematics, teachers mainly followed the order and the instructions of the mathematics books. The right answers were more important than solution procedures.

Next I restrict my attention to two cases, namely of Bertta and Cecilia. In Table 1 you can see that Cecilia’s beliefs are primarily non traditional but the beliefs of Bertta are nearly mixed. Both of them had non-traditional features in their beliefs. Bertta has been a teacher for over twenty years and Cecilia has been a teacher only one year. Cecilia had studied a teacher training programme with new learning and teaching methods. Cecilia had also more mathematics studies than Bertta. In spite of the differences in their beliefs, their teacher training programme and their teaching experience, they were both dependent on mathematics textbooks and they felt uncertainty with mathematics. Their teaching practices were more close to traditional (instrumentalist) view than non-traditional view (problem-solving view).
The case of Bertta

The mathematical history of Bertta: At her school-time she had had difficulties with mathematics in mental arithmetic and word problems. She felt that she was rather poor at mathematics. During the interview Bertta explained her recollections about mathematics studies at school:

“I like to teach mathematics. Very often I am recollecting how I had difficulties with story problems and mental arithmetic in mathematics in the third and the fourth grade in primary school. I remember that I was very poor at mathematics. That is why mathematics teaching is a challenge to me and it is very important for me to do mathematics teaching practice meaningful to these children.”

Bertta did not remember anything special about the mathematics studies during her teacher education. Her teaching philosophy is as follows:

“I think it is learning mathematics by doing it through activities and also co-operative learning. I think that if you put children who have difficulties with mathematics together with those who are good at mathematics, they can help each other. Those who know story problems can teach those who do not know story problems”.

Bertta taught the first grade. According to the questionnaire, Bertta’s conceptions about mathematics were rules and strict orders but also problem solving and creativity. On the basis of Bertta’s belief questionnaire answers, learning mathematics meant for her developing children’s creativity, concrete learning environments and also confidence in the power of mathematics textbooks and their complete patterns; it seemed that those patterns had formed some kind of limits for her on how to learn mathematics. Bertta’s answers about mathematics teaching emphasized understanding through problem solving and the use of manipulatives, children’s own solutions but also mathematics exactness and rote learning and routines.

There were inconsistencies between Bertta’s beliefs (belief questionnaire answers) and teaching practices. Especially the story problems had been difficult for her and she wanted that her pupils should learn mathematics properly. During the classroom observations I found that the mathematics textbooks were sometimes very dominating. For example Bertta was so dependent on the textbooks that she did not hear the child’s right explanation of the right solution or she did not give any space to children’s thinking (inconsistency with children’s creativity) - she only waited for the right answers or sentences that should be filled in the textbooks. The right answers were more important than the children’s thinking. After the classroom observation Bertta analysed her teaching and her dependence on the mathematics textbooks like this:

“Here my teaching is teacher-directed because I thought that you should see how I teach this addition over ten. If I had had more time I could have used manipulatives and co-operative working. ....Because the children can’t read so well in the first grade we have to do these story problems together. It is also important
to give children good experiences in mathematics and by doing together the story problems everyone can calculate them. There are still those who can do these problems alone.“

Comments: Although the children were sometimes encouraged to work together and had activities (mathematics is fun), I found that the mathematics textbooks were very dominating. All her lessons were textbook-centred. Because of her own difficulties with mathematics during her school-time she wanted her pupils to learn mathematics properly and avoid the same kind of difficulties with mathematics she had had. She wanted to confirm that children could do the textbook problems. That is why she always counted the different problems together with children. So, she did not give any space to children’s own thinking.

The Case of Cecilia

The mathematical history of Cecilia: Cecilia’s recollections about her school-time were following:

“I have always liked math because I calculated mathematics exercises quickly. We had textbooks at school. We looked at the formula and then we applied it. That was that. Sometimes I had difficulties with story problems and that is why I think I can understand those children who have difficulties with mathematics. I try to give those children manipulatives but it takes so much time from the lessons.“

In spite of the fact that Cecilia specialized in mathematics she felt that her mathematics studies were not good enough during her teacher training programme. She felt that her mathematics education did not give her readiness to teach mathematics.

Cecilia’s beliefs (on the basis of belief questionnaire answers) about mathematics stressed the problem solving view and creativity. She thought that mathematics teaching should be creative and children should do mathematics and give their own ideas. I think that these beliefs were from her teacher training programme.

During the classroom observations I found that her teaching practices were often textbook-centred. At the beginning of the lesson Cecilia often taught children at the blackboard and after that the children calculated exercises from their textbooks. After the classroom observation Cecilia analysed her teaching like this:

“It is not so creative in mathematics. I think that geometry teaching was more creative because we had then manipulatives”.

Comments: I found that Cecilia was depending on mathematics textbooks. She told me that her lessons were textbook-centred because her teacher education did not give her so good knowledge in mathematics that she could use textbook as one of the manipulatives. Because of her uncertainty in mathematics, Cecilia mainly followed the order and the instructions of the mathematics textbook. Cecilia’s recollections about her mathematical history stressed mathematics textbooks and counting. She wanted to
teach in child-centred way but her recollections about mathematics teaching were stronger.

**Conclusion**

Berta was a very experienced teacher and Cecilia was a young teacher. In spite of their differences in teaching experience they had same kind of features in their teaching practices; for example text-book centered teaching, and calculating without manipulatives. It seemed that the effect of their school-time recollections was very strong. Also the teachers beliefs about the content of mathematics were more strongly linked to teaching practice than their beliefs about mathematics teaching and learning. So in teacher education we should pay more attention to students’ own thinking and reflection. According to Raymond (1997, 574), ‘early and continued reflection about mathematics beliefs and practices, beginning in teacher preparation, may be the key to improving the quality of mathematics instruction and minimizing inconsistencies between beliefs and practice.’ Teachers should not work in isolation either but as members of learning communities. It is important to encourage them to work and pursue new ideas together because by considering both the positive and negative consequences of various teaching practices teachers would come to a better understanding of their own beliefs and would consider whether they are consistent with their goals when teaching their pupils (Barnett 1998). In teacher training it is important to pay more attention to learning theories and their connection to the mathematics teaching and learning situations. The most important thing is not to cover the textbooks and give right answers. Instead we should understand the child and know how she or he learns. If we do not listen to children’s answers the children become uncertain and they will eventually answer what they think the teacher expects.

**References**


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